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CALCULUS OF BENDING MOMENT CORRESPONDING TO THE CENTRIFUGAL FORCE

BY

FLORENTINA MOCANU*

"Gheorghe Asachi" Technical University of Iaşi,
Department of Mechanical Engineering, Mechatronics and Robotics

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Abstract. The paper offers an original contribution for calculus of bending moment corresponding to the centrifugal force in a bar as a truncated cone form with a specific elliptical section.

Keywords: centrifugal force; bending moment; curve bar; elliptical section.

1. Introduction

The paper presents in a unitary way some specific aspects, as resistance corresponding to the centrifugal force, for elastic deformations, for a specific bar design. On considered for bar a form as a truncated cone with a specific elliptical section.

2. Centrifugal Force

2.1. Vertical Part of the Bar

The specific bar design is composed from two main parts:
– vertical part, parallel with the bar's rotation axis;

*Corresponding author; *e-mail*: florentinamoc@yahoo.com

– curved part.

The horizontal part, perpendicular on the bar's rotation axis, represents the joint element between the bar which we are analysis and other element of structure (Fig. 1). The bar has a truncated cone form with a specific elliptical section.

The expression of the elementary centrifugal force corresponding to an elementary volume separated at the distance *x* is:

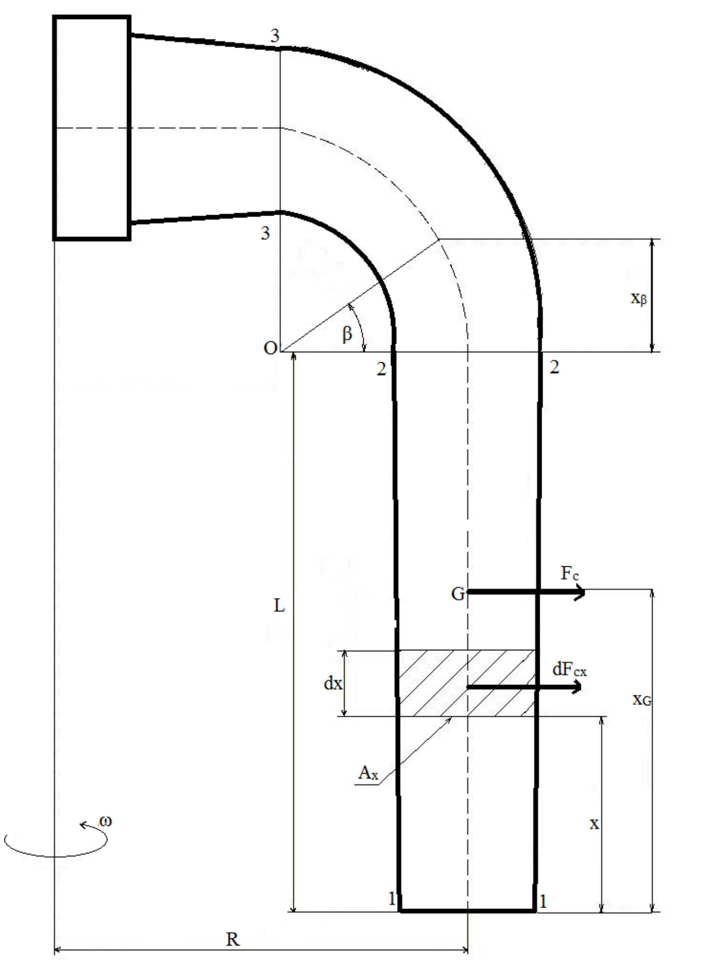

Fig. 1 – The specific bar.

$$dF_{c_x} = dm \cdot \omega^2 \cdot R = \rho \cdot dV \cdot \omega^2 \cdot R = \rho \cdot A_x \cdot dx \cdot \omega^2 \cdot R = \\ = \rho \cdot \pi \cdot a_x \cdot b_x \cdot \omega^2 \cdot R \cdot dx \tag{1}$$

where: ρ – density of the bar; ω – angular speed; A_x – transversal section area of the elementary volume; a_x, b_x – semi axes of the elliptical section; dx – thickness of the elementary volume; R – distance between the rotation axis of the bar and the arm's vertical section axis (Buzdugan, 1986).

The linear variation law of the semi axes can be written as follows:

$$a_x = a_1(1 + k_1 x)$$
$$b_x = b_1(1 + k_2 x) \tag{2}$$

with

$$k_1 = \frac{a_2 - a_1}{a_1 \cdot L}; k_2 = \frac{b_2 - b_1}{b_1 \cdot L}$$

where: a_1, a_2 – the big semi axes of the elliptical transversal section corresponding to the sections 1-1 respectively 2-2; b_1, b_2 – the small semi axes corresponding to the same ellipses; L – the length of vertical part of the bar.

By replacing the relations (2) in the relation (1) is results:

$$dF_{c_x} = \rho \cdot \pi \cdot R \cdot \omega^2 \cdot \left(Ax^2 + Bx + a_1 b_1\right) \cdot dx \tag{3}$$

The expressions for the coefficients A, B are:

$$A = \frac{1}{L^2}(a_2 - a_1)(b_2 - b_1); B = \frac{1}{L}\left[a_1(b_2 - b_1) + b_1(a_2 - a_1)\right] \tag{4}$$

The expression of centrifugal force can be determined by integrating the relation (3).

$$F_c = \int_0^L dF_{c_x} = \int_0^L \rho\pi R\omega^2\left(Ax^2 + Bx + a_1 b_1\right)dx$$

After all the calculus it results:

$$F_c = \frac{\pi L\rho\omega^2 R}{6}\left[(2a_2 + a_1)b_2 + (2a_1 + a_2)b_1\right] \tag{5}$$

2.2. Curved Part of the Bar

To establish, in a current section from the curve bar, the expression of the centrifugal force it is necessary to be isolated an infinite small element of elementary thickness. The element is obtained by sectioning on a radial direction by two successive sectioning under the angles α, respectively $\alpha + d\alpha$ from the horizontal axis (Fig.2).

It is considered the elementary volume:

$$dV = A_\alpha \cdot dS \tag{6}$$

where: A_α – the aria of the transversal section of the curve bar; dS – the elementary length of the bar portion limited by two sections.

In Eq. (6) we have:

$$dS = r \cdot d\alpha$$
$$A_\alpha = \pi \cdot a_\alpha \cdot b_\alpha \tag{7}$$

where: a_α and b_α – the large semi axis, respectively the small semi axis of the elliptical transversal section (Mocanu, 2011).

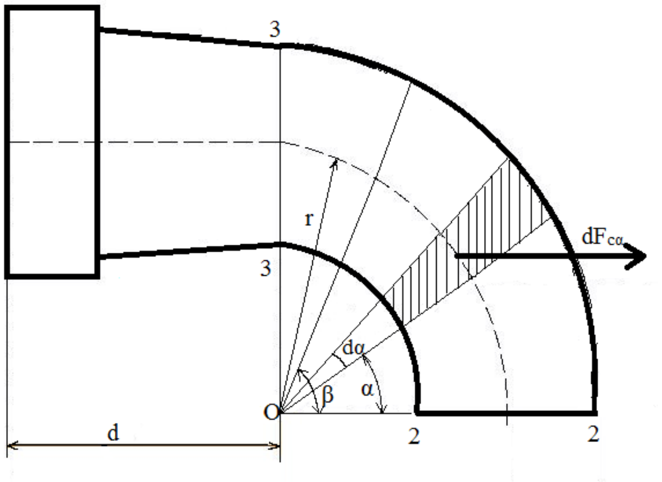

Fig. 2 – The curve bar.

The expression of the elementary centrifugal force is:

$$dF_{c_\alpha} = dm \cdot \omega^2 \cdot r_\alpha = \rho \cdot dV \cdot \omega^2 \cdot r_\alpha = \rho \cdot A_\alpha \cdot dS \cdot \omega^2 \cdot r_\alpha = \\ = \rho \cdot \pi \cdot a_\alpha \cdot b_\alpha \cdot r \cdot \omega^2 \cdot r_\alpha \cdot d\alpha \quad (8)$$

Concerning Fig. 2:

$$r_\alpha = r \cdot \cos\alpha + d \quad (9)$$

Replacing the relation (9) in Eq. (8) it results:

$$dF_{c_\alpha} = \rho \cdot \pi \cdot a_\alpha \cdot b_\alpha \cdot r \cdot \omega^2 \cdot (r \cdot \cos\alpha + d) \cdot d\alpha \quad (10)$$

The linear variation law of the semi axes can be written as follows:

$$a_\alpha = a_2(1 + k_3\alpha)$$
$$b_\alpha = b_2(1 + k_4\alpha)$$

with

$$k_3 = \frac{a_3 - a_2}{a_2} \cdot \frac{2}{\pi}$$
$$k_4 = \frac{b_3 - b_2}{b_2} \cdot \frac{2}{\pi}$$

where: a_2, a_3 – the large semi axis, b_2, b_3 – the small semi axes of the elliptical transverse section corresponding to the sections 2-2 respectively 3-3.
By replacing the last relations in Eq. (10) it is obtained:

$$dF_{c_\alpha} = \rho \cdot \pi \cdot r \cdot \omega^2 \cdot \left(C\alpha^2 + D\alpha + a_2 b_2\right)(r \cdot \cos\alpha + d) \cdot d\alpha \quad (11)$$

Where C, D are calculated by the expressions:

$$C=\frac{4}{\pi^2}(a_3-a_2)(b_3-b_2);D=\frac{2}{\pi}\left[a_2(b_3-b_2)+b_2(a_3-a_2)\right] \tag{12}$$

2.3. Bending Moment to the Centrifugal Force

In a current section from the curve bar (β section) the bending moment corresponding to the centrifugal force can be written as having two components:

$$M(\beta)=M_1(\beta)+M_2(\beta) \tag{13}$$

where: $M_1(\beta)$ – the bending moment given by the centrifugal force corresponding to the vertical part of the bar; $M_2(\beta)$ – the bending moment corresponding to the centrifugal force on the curve part of the bar.

The bending moment given by the centrifugal force corresponding to the vertical part of the bar can be calculated as:

$$M_1(\beta)=F_c\cdot\left[(L-x_G)+x_\beta\right] \tag{14}$$

where: F_c – the centrifugal force corresponding to the vertical part of the bar given by Eq. (5); x_G – the coordinate of the center weight; x_β – the vertical distance from section β to the section 2-2 (Fig. 1) (Mocanu, 2011).

For the calculus of the position of the center weight of the vertical part of the bar it is used the relation:

$$x_G=\frac{\int_0^L dV_x}{V} \tag{15}$$

where: dV_x – the elementary volume obtained by successive sectioning at the distances x and $x+dx$; V – the volume of the entire vertical part of the bar (the volume of a truncated cone with an elliptical section).

But:

$$dV_x=A_x\cdot dx=\pi a_x b_x\cdot dx=\pi a_1(1+k_1x)b_1(1+k_2x)dx=\pi\left(Ax^2+Bx+a_1b_1\right)dx \tag{16}$$

where A, B are calculated with the relations (4).

On the other hand:

$$V=\frac{\pi L}{6}\left[(2a_2+a_1)b_2+(2a_1+a_2)b_1\right] \tag{17}$$

Replacing the relations (16) and (17) in relation (15) and doing the integration and all the necessary calculus it is obtained:

$$x_G=\frac{L}{2}\cdot\frac{3a_2b_2+a_1b_2+a_2b_1+a_1b_1}{2a_2b_2+a_1b_2+a_2b_1+2a_1b_1} \tag{18}$$

Concerning Fig. 2 the vertical distance from section β to the section 2-2 can be geometrically expressed as follows:

$$x_\beta = r\sin\beta \tag{19}$$

Is replaced the relations (5), (18) and (19) in relation (14). After all the necessary calculations it is obtained the expression of the bending moment under the next form:

$$M_1(\beta) = \frac{\pi L^2}{12}\rho\omega^2 R\cdot(\Gamma + \Delta\sin\beta) \tag{20}$$

with:
$$\Gamma = a_2b_2 + a_1b_2 + a_2b_1 + 3a_1b_1$$
$$\Delta = \frac{2r}{L}\left[(2a_2 + a_1)b_2 + (2a_1 + a_2)b_1\right]$$

The bending moment of the elementary centrifugal forces in β section ($\beta \in \left[0, \frac{\pi}{2}\right]$) on the curve part of the bar has the expression (Mocanu, 2011):

$$dM_2(\beta) = dF_{c_\alpha}\cdot br \tag{21}$$

In relation (21) *br* is the arm of the elementary centrifugal force. Concerning Fig. 2 this arm can be geometrically expressed as follows:

$$br = r\sin\beta - r\sin\alpha = r(\sin\beta - \sin\alpha) \tag{22}$$

By replacing in Eq. (21) the relations (11) and (22) it is obtained:

$$dM_2(\beta) = \rho\pi\cdot r^2\omega^2\left(C\alpha^2 + D\alpha + a_2b_2\right)\cdot(r\cos\alpha + d)\cdot(\sin\beta - \sin\alpha)d\alpha$$

The bending moment corresponding to the centrifugal force on the curve part of the bar can be written follows:

$$M_2(\beta) = \int_0^\beta dM(\beta) = \rho\pi\cdot r^2\omega^2\int_0^\beta\left(C\alpha^2 + D\alpha + a_2b_2\right)(r\cos\alpha + d)(\sin\beta - \sin\alpha)d\alpha$$

After integration the last relation and executing all calculus it is obtained:

$$M_2(\beta) = \rho\pi\cdot r^3\omega^2\cdot\Omega + \rho\pi\cdot r^3\omega^2\cdot\Psi + \rho\pi\cdot r^2\omega^2 d\cdot\Delta \tag{23}$$

with:

$$\Omega = \frac{a_2b_2}{2}\sin^2\beta$$

$$\Psi = \sin^2\beta\left(\frac{C}{2}\beta^2 + \frac{D}{2}\beta\right) + \sin 2\beta\left(\frac{3}{4}C\beta + \frac{3}{8}D\right) -$$
$$- D\sin\beta + \frac{1}{4}C\beta^2 + C\left(\beta^2\cos\beta - 2\beta\sin\beta - 2\cos\beta + 2\right)$$

$$\Delta = \left(\frac{C}{3}\beta^3\sin\beta + \frac{D}{2}\beta^2\sin\beta\right) + a_2b_2\sin\beta + D\beta\cos\beta -$$
$$- D\sin\beta + a_2b_2\cos\beta - a_2b_2$$

In β section the bending moment of the centrifugal forces $M(\beta)$ is obtained by adding the relations (20) and (23).

Section 3-3 is dangerous section for the studied bar. The maximum bending moment could be determined by particularization of the expression of $M(\beta)$. After replacement and performing all calculations is obtained the maximum value of the moment:

$$\begin{aligned} M_{\max} &= M\left(\beta=\frac{\pi}{2}\right)=\frac{\pi L^2}{12}\rho\omega^2 R\left(a_2b_2+a_1b_2+a_2b_1+3a_1b_1\right)+ \\ &+\frac{\pi L}{6}\rho\omega^2 Rr\left[\left(2\mathrm{a}_2+a_1\right)b_2+\left(2\mathrm{a}_1+a_2\right)b_1\right]+\frac{\rho\pi r^3\omega^2 a_2b_2}{2}+ \\ &+\rho\pi r^3\omega^2\left\{0.363\left[a_2\left(b_3-b_2\right)+b_2\left(a_3-a_2\right)\right]-0.425\left(a_3-a_2\right)\left(b_3-b_2\right)\right\}+ \\ &+\rho\pi r^2 d\omega^2\left\{0.52\left[a_2\left(b_3-b_2\right)+b_2\left(a_3-a_2\right)\right]+0.15\left(a_3-a_2\right)\left(b_3-b_2\right)+0.57a_2b_2\right\} \end{aligned} \tag{24}$$

It takes into account the particular situation when the transversal section of the bar is constant (an elliptical section with a and b large semi axis, respectively small semi axis). In this case the maximum value of the moment is obtained by the particularization of the last relation and has the next expression:

$$M'_{\max}=\frac{\pi ab}{2}\rho\omega^2\left[LR\left(L+2r\right)+r^3+1.14dr^2\right] \tag{25}$$

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CALCULUL MOMENTULUI ÎNCOVOIETOR CORESPUNZĂTOR FORŢEI CENTRIFUGE

(Rezumat)

Lucrarea prezintă o abordare originală privind calculul momentului încovoitor corespunzător forţei centrifuge. Calculul momentului s-a efectuat pentru bara cu o formă specifică formată dintr-o porţiune dreaptă, verticală, paralelă cu axa de rotaţie şi o porţiune curbă sub formă de arc de cerc. Se consideră forma barei ca fiind cea a unui trunchi de con cu o secţiune transversală eliptică, variabilă pe lungimea barei. S-a stabilit expresia momentului încovoietor într-o secţiune curentă a barei, dar şi valoarea maximă a acestuia în secţiunea periculoasă. Prin particularizare s-a determinat valoarea maximă a momentului încovoietor dacă secţiunea transversală a barei este constantă şi are formă eliptică.